Transient Temperatures in Laminated Composite Conical Shells **Due to Aerodynamic Heating**

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Abstract

AMINATED composite conical shells are extensively Jused as nose cones in many aerospace applications because of a variety of engineering features, such as electromagnetic transparency, good heat insulation, high specific strength and stiffness, etc. One of the problems faced in designing these nose cones is their response to aerodynamic heating. Thus, transient heat conduction analysis of laminated composite media has been of interest to researchers in the field. In this paper, transient temperatures in laminated composite conical shells subjected to aerodynamic heating are presented. Unsteady heat conduction equations for a laminated composite conical shell corresponding to an axisymmetric temperature field are formulated. The edge conditions imposed are one edge at a prescribed temperature and the other edge insulated. The outer surface of the conical shell is subjected to heat transfer through the boundary layer whereas the inner surface is assumed to be insulated. The governing equations are solved by Galerkin's technique in conjunction with the Laplace transform technique, and temperatures in each layer are obtained as a function of generatrix coordinate and time. In this analysis, boundarylayer temperature and heat-transfer coefficient are assumed to be time independent. However, the results are used for a realistic situation where these quantities vary with time. This is done by assuming that the shell is subjected to a heat input corresponding to the maximum values of heat-transfer coefficient and boundary-layer temperature. These assumptions would imply a conservative estimate of the temperature field in the shell.

Content

The Fourier heat conduction equation for the *i*th anisotropic layer of a composite conical shell for an axisymmetric temperature field is written as follows:

$$\rho c \frac{\partial T_i}{\partial t} = K_{ss}^i \frac{\partial}{\partial s} \left(s \frac{\partial T_i}{\partial s} \right) + k_{zz}^i \frac{\partial^2 T_i}{\partial z^2} \tag{1}$$

where T_i = temperature of the *i*th layer and the other symbols have their usual meaning.

In many applications, the thickness of each layer would be of the order of 0.125-0.250 mm. Hence, it is quite adequate to assume that the temperature variation across the thickness of each layer will be insignificant, and one can conveniently carry out the heat conduction analysis in terms of an average

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temperature T_i^* which is defined as follows:

$$T_i^*(s) = \frac{1}{h} \int_{-h/2}^{h/2} T_i(s) \, \mathrm{d}z \tag{2}$$

Using Eq. (2), Eq. (1) is changed to a differential equation in T_i^* . The resulting equation is then nondimensionalized. Further, introducing flux continuity conditions at the interface and surface conditions the governing heat conduction equations for the first, ith and nth layer are obtained.

The following edge conditions are enforced

$$(T_i^*)_{y=y_0} = T_{st}, \quad (T_{i,y}^*)_{y=1} = 0$$
 (3)

where T_{st} = stagnation temperature.

Now the Laplace transform technique is employed to eliminate the time variable from the governing equations. This results in a set of ordinary coupled differential equations

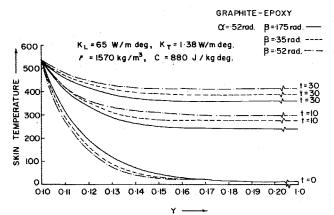


Fig. 1 Spatial variation of skin temperature (°C) at various times for graphite-epoxy shells.

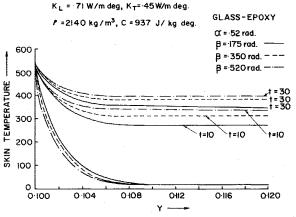


Fig. 2 Spatial variation of skin temperature (°C) at various times for glass-epoxy shells.

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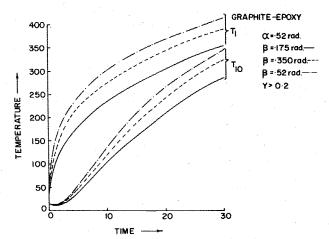


Fig. 3 Variation of uniform temperature (°C) of different layers time for graphite-epoxy shells.

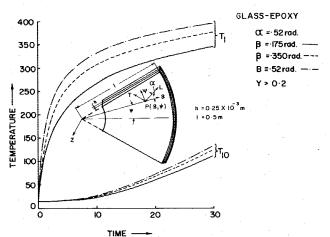


Fig. 4 Variation of uniform temperature (°C) of different layers with time for glass-epoxy shells.

with nondimensional temperatures as dependent variables and y as an independent variable. These equations are solved by employing Galerkin's technique. The temperature function satisfying the edge conditions in Eq. (3) is assumed as follows:

$$\{\bar{T}\} = (1/p)\{I\} + \{C\}\{I - \exp[-\epsilon(y - y_{\theta})]\}$$
 (4)

The justification for using the aforementioned one-term function arises from the fact that this function yields a transient temperature distribution in very close agreement with the exact one for an isotropic conical shell.¹

Substituting relation (4) in the governing equations and applying Galerkin's technique, the following matrix equation results.

$$[a]{C} + p{C} = {R} + (1/p){\ell}$$
(5)

This equation is solved in terms of p by a method based on eigenvalues and, thus, the unknown $\{C\}$ is determined. The temperature matrix $\{T\}$ is determined by using relation (4) and then taking the inverse transform of $\{\bar{T}\}$. Thus, the

temperatures in each layer are obtained as a function of time and the coordinate along the generator.

Using these results, the effect of the geometrical parameters, semivertex angle β and length-to-thickness ratio, and the lamination parameters, fiber orientation α and number of layers, on the temperature field in both a glassepoxy and graphite-epoxy shell is presented. The heat-transfer coefficient and boundary-layer temperature required for this work are taken from Ref. 2 and are given as follows: boundary-layer temperature = 485°C, heat-transfer coefficient = 600 W/m² deg for β = 0.175 rad; heat-transfer coefficient = 805 W/m² deg for β = 0.350 rad; heat-transfer coefficient = 991 W/m² deg for β = 0.520 rad.

Figures 1 and 2 represent the variation of skin temperatures along the generator of 10-layered graphite-epoxy and glass-epoxy shells, respectively. The effect of β on the temperature distribution is studied here by varying β from 0.175 rad to 0.52 rad while keeping the other parameters constant. It is seen from the relations given in Ref. 2 that the heat-transfer coefficient increases with β . Hence the effect of β is to enhance the temperatures in the shells in the transient time domain. Another interesting feature revealed by these curves is that the prescribed temperature edge condition has a localized effect on the temperature distribution in these shells and temperatures at y>0.2 are uniform along the generator. This effect is more localized in glass-epoxy shells because of their low value of thermal conductivity.

The effect of fiber orientation also was studied in this work. It was found that α does not influence the temperatures significantly in glass-epoxy shells, whereas it does influence them in graphite-epoxy shells. This is because K_L/K_T for graphite-epoxy is around 30 times that for glass-epoxy.

Figures 3 and 4 represent the effect of β on a variation of uniform temperatures with time for 10-layered graphite-epoxy and glass-epoxy shells, respectively. It is noted from these figures that significant temperature gradients exist across the thickness for both glass-epoxy and graphite-epoxy shells. Also, these temperature gradients are noticed to be more severe for glass-epoxy as compared to graphite-epoxy shells. The effect of number of layers also was studied by comparing results corresponding to 10 layers and 5 layers. As expected, the temperature gradients were found to be more severe for 10-layered shells as compared to 5-layered shells.

The transient temperature field in a laminated composite conical shell subjected to aerodynamic heating has been determined employing an analytical technique. The main contribution of this work lies in applying the Laplace transform technique to a practical structure like the laminated composite conical shell. A limitation of this work is that spatial variation of heat input can not be taken into account. An important revelation of this study is that temperature gradients across the thickness of the shell are quite significant and should be considered for further thermal stress analysis of the shell.

References

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